CHAPTER 8

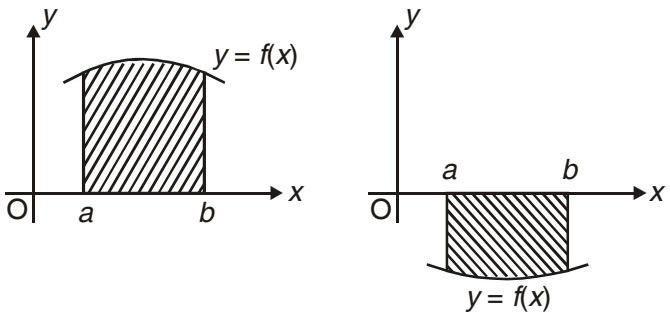
APPLICATIONS OF INTEGRALS

POINTS TO REMEMBER

AREA OF BOUNDED REGION

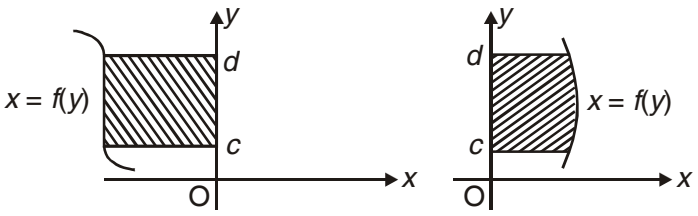
- Area bounded by the curve $y = f(x)$, the x axis and between the ordinates, $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b |f(x)|\, dx$$

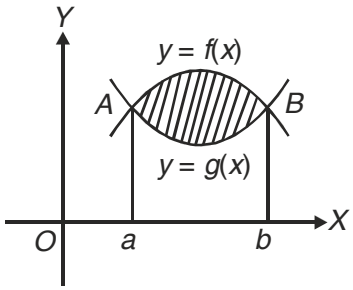


- Area bounded by the curve $x = f(y)$ the y-axis and between abscissas, $y = c$ and $y = d$ is given by

$$\text{Area} = \int_c^d |f(y)|\, dy$$



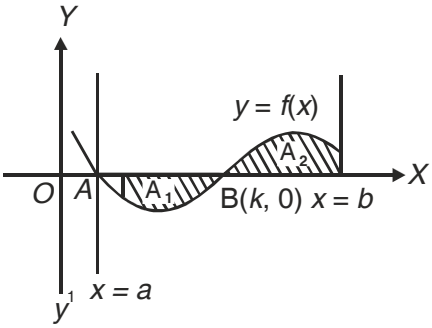
- Area bounded by two curves $y = f(x)$ and $y = g(x)$ such that $0 \le g(x) \le f(x)$ for all $x \in [a, b]$ and between the ordinate at $x = a$ and $x = b$ is given by



$$\text{Area} = \int_a^b [f(x) - g(x)]\, dx$$

- Required Area

$$= \left| \int_a^k f(x)dx \right| + \int_k^b f(x)dx.$$



LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. Find the area enclosed by circle $x^2 + y^2 = a^2$.

2. Find the area of region bounded by $\left\{(x, y) : |x - 1| \le y \le \sqrt{25 - x^2}\right\}$.

3. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

4. Find the area of region in the first quadrant enclosed by x–axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

5. Find the area of region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

6. Prove that the curve $y = x^2$ and, $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$ into three equal parts.

7. Find smaller of the two areas enclosed between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line

   $bx + ay = ab$.

8. Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

9. Using integration, find the area of the region bounded by the triangle whose vertices are

   (a) (–1, 0), (1, 3) and (3, 2) (b) (–2, 2) (0, 5) and (3, 2)

10. Using integration, find the area bounded by the lines.

    (i) $x + 2y = 2$, $y - x = 1$ and $2x + y - 7 = 0$

    (ii) $y = 4x + 5$, $y = 5 - x$ and $4y - x = 5$.

11. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

12. Find the area of the region bounded by

    $y = |x - 1|$ and $y = 1$.

13. Find the area enclosed by the curve $y = \sin x$ between $x = 0$ and $x = \frac{3\pi}{2}$ and x-axis.

14. Find the area bounded by semi circle $y = \sqrt{25 - x^2}$ and x-axis.

15. Find area of region given by $\{(x, y) : x^2 \leq y \leq |x|\}$.

16. Find area of smaller region bounded by ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and straight line $2x + 3y = 6$.

17. Find the area of region bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$.

18. Using integration find the area of region in first quadrant enclosed by x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

19. Find smaller of two areas bounded by the curve $y = |x|$ and $x^2 + y^2 = 8$.

20. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

21. Using integration, find the area enclosed by the curve $y = \cos x$, $y = \sin x$ and x-axis in the interval $\left(0, \frac{\pi}{2}\right)$.

22. Sketch the graph $y = |x - 5|$. Evaluate $\int_0^6 |x - 5|\, dx$.

23. Find area enclosed between the curves, $y = 4x$ and $x^2 = 6y$.

24. Using integration, find the area of the following region :

$$\left\{(x, y) : |x - 1| \le y \le \sqrt{5 - x^2}\right\}$$

ANSWERS

1. πa^2 sq. units.

2. $\left(25\frac{\pi}{4} - \frac{1}{2}\right)$ *sq. units.*

3. πab sq. units

4. $(4\pi - 8)$ sq. units

5. $\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right)$ sq. units

7. $\frac{(\pi - 2)\,ab}{4}$ sq. units

8. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units

9. (a) 4 sq. units (b) 2 sq. units

10. (a) 6 sq. unit [**Hint.** Coordinate of vertices are (0, 1) (2, 3) (4, – 1)]

(b) $\frac{15}{2}$ sq. **Hint :** Coordinate of vertices are (– 1, 1) (0, 5) (3, 2)]

11. $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq. units

12. 1 sq. units

13. 3 sq. units

14. $\frac{25}{2}\pi$ sq. units

15. $\frac{1}{3}$ sq. units

16. $\frac{3}{2}(\pi - 2)$ sq. units

17. $\frac{9}{8}$ sq. units

18. $\frac{\pi}{3}$ sq. unit

19. 2π sq. unit.

20. $\frac{4}{3}(8 + 3\pi)$ sq. units

21. $\left(2 - \sqrt{2}\right)$ sq. units.

22. 13 sq. units.

23. 8 sq. units.

24. $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units