CHAPTER 12

LINEAR PROGRAMMING

POINTS TO REMEMBER

● Linear programming is the process used to obtain minimum or maximum value of the linear objective function under known linear constraints.

● **Objective Functions :** Linear function $z = ax + by$ where a and b are constants, which has to be maximized or minimized is called a linear objective function.

● **Constraints :** The linear inequalities or inequations or restrictions on the variables of a linear programming problem.

● **Feasible Region :** It is defined as a set of points which satisfy all the constraints.

● **To Find Feasible Region :** Draw the graph of all the linear inequations and shade common region determined by all the constraints.

● **Feasible Solutions :** Points within and on the boundary of the feasible region represents feasible solutions of the constraints.

● **Optimal Feasible Solution :** Feasible solution which optimizes the objective function is called optimal feasible solution.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. Solve the following L.P.P. graphically

    Minimise and maximise $z = 3x + 9y$

    Subject to the constraints $x + 3y \leq 60$

    $x + y \geq 10$

    $x \leq y$

    $x \geq 0,\ y \geq 0$

2. Determine graphically the minimum value of the objective function $z = -50x + 20y$, subject to the constraints

$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0, \ y \geq 0$$

3. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. Formulate the above L.P.P. mathematically and hence solve it to minimise the labour cost to produce at least 60 shirts and 32 pants.

4. There are two types of fertilisers A and B. A consists of 10% nitrogen and 6% phosphoric acid and B consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If A costs Rs. 61 kg and B costs Rs. 51 kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at minimum cost. What is the minimum cost?

5. A man has Rs. 1500 to purchase two types of shares of two different companies S_1 and S_2. Market price of one share of S_1 is Rs 180 and S_2 is Rs. 120. He wishes to purchase a maximum of ten shares only. If one share of type S_1 gives a yield of Rs. 11 and of type S_2 yields Rs. 8 then how much shares of each type must be purchased to get maximum profit? And what will be the maximum profit?

6. A company manufactures two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finishers has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp B is Rs. 13.00. Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit?

7. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for almost 20 items. A fan and sewing machine cost Rs. 360 and Rs. 240 respectively. He can sell a fan

at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest his money to maximise his profit?

8. If a young man rides his motorcycle at 25 km/h, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as L.P.P. and then solve it graphically.

9. A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of X, 2 units of capital and 1 unit of labour is required. To produce one unit of Y, 3 units of labour and one unit of capital is required. If X and Y are priced at Rs. 80 and Rs. 100 per unit respectively, how should the producer use his resources to maximise the total revenue?

10. A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows:

Machine	Area Occupied	Labour Force	Daily Output (In units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output.

11. A manufacturer makes two types of cups A and B. Three machines are required to manufacture the cups and the time in minutes required by each in as given below :

Types of Cup	Machine		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise and on B is 50 paise, find how many cups of each type should be manufactured to maximise the profit per day.

12. A company produces two types of belts *A* and *B*. Profits on these belts are Rs. 2 and Rs. 1.50 per belt respectively. A belt of type *A* requires twice as much time as belt of type *B*. The company can produce almost 1000 belts of type *B* per day. Material for 800 belts per day is available. Almost 400 buckles for belts of type *A* and 700 for type *B* are available per day. How much belts of each type should the company produce so as to maximize the profit?

13. Two Godowns *X* and *Y* have a grain storage capacity of 100 quintals and 50 quintals respectively. Their supply goes to three ration shop *A*, *B* and *C* whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintals from the godowns to the shops are given in following table :

To From	Cost of transportation (in Rs. per quintal)	
	X	Y
A	6.00	4.00
B	3.00	2.00
C	2.50	3.00

How should the supplies be transported to ration shops from godowns to minimize the transportation cost?

14. An Aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However atleast four times as many passengers prefer to travel by second class than by first class. Determine, how many tickets of each type must be sold to maximize profit for the airline.

15. A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods *A* and *B* are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of food *A* contains 200 unit of vitamins, 1 unit of minerals and 40 units of calories whereas one unit of food *B* contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food *A* and *B* should be used to have least cost but it must satisfy the requirements of the sick person.

ANSWERS

1. Min $z = 60$ at $x = 5$, $y = 5$.

    Max $z = 180$ at the two corner points $(0, 20)$ and $(15, 15)$.

2. No minimum value.

3. Minimum cost = Rs. 1350 at 5 days of A and 3 days of B.

4. 100 kg. of fertiliser A and 80 kg of fertilisers B; minimum cost *Rs.* 1000.

5. Maximum Profit = Rs. 95 with 5 shares of each type.

6. Lamps of type A = 40, Lamps of type B = 20.

7. Fan : 8; Sewing machine : 12, Max. Profit = Rs. 392.

8. At 25 km/h he should travel 50/3 km, At 40 km/h, 40/3 km. Max. distance 30 km in 1 hr.

9. X : 2 units; Y : 6 units; Maximum revenue Rs. 760.

10. Type A : 6;   Type B : 0

11. Cup A : 15;   Cup B : 30

12. Maximum profit Rs. 1300, No. of belts of type A = 200 No. of bells of type B = 600.

13. From X to A, B and C : 10 quintals, 50 quintals and 40 quintals respectively.

    From Y to A, B, C : 50 quintals, NIL and NIL respectively.

14. No. of first class tickets = 40, No. of 2nd class tickets = 160.

15. Food A : 5 units, Food B : 30 units.

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